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A Study of Structure Functions for the Bag Beyond Leading Order

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Abstract

There has recently been surprising progress in understanding the spin and flavor dependence of deep inelastic structure functions in terms of the same physics needed in the simple quark models used for hadronic spectroscopy. However, the corresponding scale is usually very low, casting doubt on the use of leading order QCD evolution. We show that the conclusions are not significantly altered if one goes to next-to-leading order. In particular, the excellent agreement with unpolarized and polarized valence quark distributions is retained.

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1   Introduction

Deep inelastic scattering continues to provide a wealth of surprising and challenging information concerning the structure of nucleons and nuclei. The nuclear EMC effect, now more than a decade old [1], provided a dramatic challenge to our conventional view of nuclear structure [2]. In particular, it suggested a systematic change in the valence quark structure of a nucleon inside matter. This clearly goes to the heart of our understanting of nuclear structure. More recently the European Muon Collaboration revealed [3] an unexpected deviation from the Ellis-Jaffe sum rule [4], which became known as the "spin crisis". Finally the New Muon Collaboration [5] has confirmed suggestions [6] of a violation of SU(2) flavour symmetry in the nucleon sea.

All of these observations demand theoretical interpretation. In the absence of reliable calculations using lattice QCD, and with QCD sum rules yielding only a few moments, one is naturally led to the problem of relating low energy quark models to the parton distributions measured in deep-inelastic scattering. Given that deep inelastic data is valence dominated at low-Q^2, it was suggested fairly early [7] that the natural connection was to use the quark model to calculate the leading twist parton distributions at some low scale, μ^2, and to evolve them to a higher scale, Q^2, using the renormalization group equations (RGE) and then compare the resulting distributions with data [7]. In the context of the bag model, Jaffe [8] and Le Yaouanc et al. [9] were the first to implement this idea, followed by Hughes [10] and Bell [11] and others [12].

A major problem in these calculations was the lack of correct support for the parton distributions.(A quark distribution has to vanish outside the region $0 \le x \le 1$.) This disease was cured some time ago in a series of papers by Signal, Thomas and Schreiber [13, 14]. The proposal was to construct the parton distributions using the formal analysis of Jaffe [15], and to ensure energy-momentum conservation before any approximations were made. (To better appreciate the importance of correct support in the distributions, the reader is referred to ref. [16].) In ref. [14] a detailed study of the MIT quark distributions was made. Various distributions, polarized and unpolarized, were calculated at the model scale and then evolved in leading order (LO), using the RGE, to the scale of the experiment. A very good, qualitative agreement was obtained, which was rewarding and stimulating given the simplicity of the model. It became clear that high energy experiments could

be, to a large extent, explained using as input low energy physics.

Of course, bag model calculations using the method of ref. [14] are not the only ones that can be found in the literature (see, for example, ref. [17]). Moreover, there are calculations based on other models. For example, there are works on the colour-dieletric model [18] or on the non-relativistic quark model [16, 19]. However, besides the fact that some of the calculations suffer from the problem of poor support, it is known [16] that the parton distributions constructed from Gaussian wave functions do not have the right behavior as x approaches 1. Recently, there have also been attempts to formulate the problem in terms of relativistic quark-nucleon vertex functions [20, 21].

Thus far, all analyses of the x dependence of parton distributions within quark models have been performed only in leading order. There are numerous studies of higher order evolution of unpolarized data in the form of parametrizations [22, 23], or higher order corrections to the first moment of some QCD sum rules [24], but no study of models involving both polarized and unpolarized data [1]. Besides the fact that corrections of higher order in the strong coupling constant $\alpha_s(Q^2)$ are already important per se, we emphasise that a next-to-leading order (NLO) analysis is necessary because the scale of the quark model is generally very low so that the applicability of the leading order RGE is questionable. In this paper, we will apply the NLO evolution to the polarized and unpolarized valence distributions in the proton and to the x dependence of the Bjorken sum rule. We will compare the resulting distribution with the MRS [23] parametrizations and with preliminary data for the polarization of valence quarks in the proton [26]. The paper is organized as follows. In Sec. II we briefly review the process of QCD evolution in NLO. In Sec. III we review the procedure for calculating quark distributions in the bag model. In Sec. IV we present some numerical results, while Sec. V is used to present our conclusions.

2   Distributions at Next to Leading Order

It is well known that one can write the moments of the structure functions as:

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[1]In ref.[25] the x dependence of $g_{1p}(x)$ was calculated in NLO. However, parametrizations of the data were used for the input

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$$M_n(Q^2) = \sum_i C_n^i(Q^2/\mu^2, g)A_n^i(\mu^2). \tag{1}$$

The sum runs over all spin-n, twist-2 operators. In this work we will restrict ourselves to the nonsinglet sector because the complete, singlet anomalous dimensions in two loops for polarized scattering are not known. (Up to now, only the quark-quark and the quark-gluon part of the singlet Altarelli-Parisi splitting functions have been calculated [25].)

Equation (1) is a direct result of the operator product expansion applied to the forward scattering of the photon from the hadronic state through the e.m. current, J_μ. The amplitudes $A_n^i(\mu^2)$ are target dependent and involve non-perturbative QCD. They are related to the moments of the quark distribution in the target at the renormalization scale μ^2. The Wilson coefficients, $C_n^i(Q^2/\mu^2, g)$, are target independent and calculable in perturbation theory. They carry all the information about the scale dependence of the structure functions and are well determined. The evolution in Q^2 for the Wilson coefficients is given by the solution of the corresponding renormalization group equation which has the form:

$$C_n(Q^2/\mu^2, g) = C_n(1, \overline{g}^2)exp\left(-\int_{\overline{g}(\mu^2)}^{\overline{g}(Q^2)} dg' \frac{\gamma^n(g')}{\beta(g')}\right). \tag{2}$$

Here, $\overline{g}(Q^2)$ is the running coupling constant (such that $\overline{g}(Q^2 = \mu^2) = g$), $\gamma^n(g)$ is the anomalous dimension of the corresponding nonsinglet operator and $\beta(g)$ is the QCD beta function. Expanding to second order they are expressed as:

$$\beta(g) = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}, \tag{3}$$

$$\gamma^n(Q^2) = \gamma^{(0)n} \frac{g^2}{16\pi^2} + \gamma^{(1)n} \frac{g^4}{(16\pi^2)^2}, \tag{4}$$

$$C^n = 1 + F^n \frac{g^2}{16\pi^2}, \tag{5}$$

while the running coupling constant is determined by solving the transcendental equation:

$$ln\frac{Q^2}{\Lambda^2} = \frac{16\pi^2}{\beta_0 \overline{g}^2} - \frac{\beta_1}{\beta_0^2}ln\left[\frac{16\pi^2}{\beta_0 \overline{g}^2} + \frac{\beta_1}{\beta_0^2}\right]. \tag{6}$$

The QCD scale parameter, Λ, is determined by comparing the theoretical calculations with the experimental data, and β_0, β_1, $\gamma^{(0)n}$, $\gamma^{(1)n}$ and F^n are parameters determined in perturbation theory. Their form is listed in the appendix. Throughout this paper we will use $\Lambda = 0.2 \, GeV$ (which is within the experimental errors [27]). For unpolarized scattering both the leading order (LO) [28] and next to leading order (NLO) [29] coefficients are well known. One need only be careful with the renormalization scheme dependence: although β_0, β_1 and $\gamma^{(0)}$ do not depend on the scheme, $\gamma^{(1)n}$ and F^n do, and both have to be calculated in the same scheme in order to obtain physically meaningful results [30]. For the nonsinglet operators, all coefficients but F^n are the same for both polarized and unpolarized scattering. The Wilson coefficient F_n was first calculated in the $\overline{MS}$ scheme by J. Kodaira and collaborators [31]. With this in mind we write the NLO evolution equation for the moments with the help of expressions (2)-(5):

$$
\begin{aligned}
M_n(Q^2) \;=\; & A_n(\mu^2) \left[\frac{\overline{g}^2(Q^2)}{\overline{g}^2(\mu^2)} \right]^{\gamma^{(0)n}/2\beta_0} \\
& \left(1 + \frac{\overline{g}^2(Q^2)}{16\pi^2} F^n + \frac{(\overline{g}^2(Q^2) - \overline{g}^2(\mu^2))}{16\pi^2} \left(\frac{\gamma^{(1)n}}{2\beta_0} - \frac{\beta_1 \gamma^{(0)n}}{2\beta_0^2} \right) \right).
\end{aligned} \tag{7}
$$

This equation can be rewritten as:

$$
M_n(Q^2) = A_n(Q^2) C_n(1, \overline{g}(Q^2)), \tag{8}
$$

which makes it clear that the moments of the quark distributions in NLO have the same Q^2 dependence as in LO, but that this is not the same for the moments of the structure functions. In other words, $A_n(Q^2)$ gives the moments of the quark distributions and $M_n(Q^2)$ gives the moments of the structure functions. Expression (8) also expresses clearly the scheme dependence of the quark distributions beyond leading order, which means that they are unphysical. From eq. (7) is straightforward to verify that in NLO, QCD sum rules, like the Bjorken sum rule, pick up $\overline{g}^2$ corrections.

3   The Quark Distributions for the Bag Model

The matrix elements $A_n(\mu^2)$ can be written in terms of parton distributions, as showed by Jaffe [32]. This is done by defining the distribution as the

integral, in the light cone gauge, of the forward virtual quark-target scattering amplitude over all the parton momenta (in the light cone) but keeping the plus component fixed and equal to xp^+ (with x the Bjorken variable and p^+ the plus component of the target momentum). The parton distribution written in this way has support only for $-1 \leq x \leq 1$. Explicitly,

$$q(x) = p^+ \sum_n \delta(p^+(1-x) - p_n^+) \mid \langle n | \Psi_+(0) | p \rangle \mid^2, \tag{9}$$

and

$$A_n = \int_{-\infty}^{+\infty} q(x) x^{n-1} dx, \tag{10}$$

where p_n is the momentum carried by the spectators to the interaction (intermediate states). An equation very similar to equation (9) can be written for the antiquark distribution, but with $\Psi^\dagger$ replacing Ψ. There are two kind of contributions to (10): one coming from the annihilation of one of the quarks in the nucleon and the other from the creation of a quark or antiquark in the nucleon. The first process is what we call the two quark contribution and it is the dominant one. We show this contribution within the model, in question, in detail below (see equ. (12)). The second contribution is a four quark one. As will become clear below, this kind of contribution will be used to fix the normalization problem of the quark distributions in the bag but it will not be calculated explicitly within the model.

The approach followed in this paper to calculate $q(x)$ will be that developed by the group at the University of Adelaide [13, 14], in which the MIT [33] bag model wave functions are used and the correct support for the distribution is fully assured. Basically, what is done is to project the moment vectors $|p\rangle$ in the coordinate space and calculate these overlaps using the Peierls-Yoccoz aproach [34]. This ensures that the wave functions are momentum eigenstates, so attacking one of the major problems of the bag model, that is the lack of translational invariance. The Peierls-Yoccoz projection is regulated by the weight functions $\phi(\vec{p})$ given by:

$$\mid \phi_l(\vec{p}) \mid^2 = \int d\vec{x} e^{-i\vec{p}\cdot\vec{x}} \left[\int d\vec{y} \psi^\dagger(\vec{y} - \vec{x}) \psi(\vec{y}) \right]^l. \tag{11}$$

Inserting a complete set of position states, converting the sum into an integral over the momentum of the intermediate states and using the projection given

above, we get for the two quark piece (full details can be found in ref. [14]):

$$
\begin{aligned}
q_f^{\uparrow\downarrow}(x) &= \frac{M}{(2\pi)^2} \sum_m \langle \mu | P_{f,m} | \mu \rangle \\
&\times \int_{[M^2(1-x)^2 - M_n^2]/2M(1-x)}^{+\infty} | \vec{p}_n | \, d \, | \vec{p}_n | \, \frac{|\phi_2(\vec{p}_n)|^2}{|\phi_3(0)|^2} |\tilde{\psi}_m^{\uparrow\downarrow}(\vec{p}_n)|^2. \quad (12)
\end{aligned}
$$

Here $|\mu\rangle$ is the spin-flavor part of the wave function of the initial state (at rest), $P_{f,m}$ makes the projection onto flavor f and spin projection m, M_n is the mass of the intermediate state and $\tilde{\psi}$ the Fourier transform of ψ. The expression for the four quark contribution has the same form, but ϕ_2 is replaced by ϕ_4.

Once we have a model to calculate the matrix elements A_n we can proceed to evalute the moments of the structure functions. Before doing so, a few remarks need to be made. The first deals with the spin of the intermediate states. In the case of a two-quark spectator system, the sum of the spins may be zero (a scalar diquark with a lower mass M_s), or one (a vector diquark with a higher mass M_v). The mass difference between these states is found to be around $200 MeV$ in most quark models, and this is crucial to understanding the soin and flavour dependence of the distributions [35]. Taking this into account, one can calculate the contributions to the quark distribution (12) due to the scalar and vector diquark systems, by using the $SU(6)$ wave function for $|\mu\rangle$. The second issue is the form of the four quark contibution. It is known [14] that the bag model has problems to fix the normalization of the quark distributions. Probably this is due the fact that we do not calculate the masses of the intermediate states in a way consistent with the model. To avoid this problem, we choose the four quark term to have the form $(1-x)^7$ and normalize it in a way that the overall quark distribution satisfies the normalization condition. Although this procedure is clearly phenomenological and not directly related to the model, we note that a term of this form will affect only the region $x < 0.3$. Furthermore its shape resembles very much the actual form of the the four quark term calculated using the same formalism as that used for the two quark contribution.

4 Results

Now that we know how to calculate the matrix elements A_n we can proceed to apply the evolution equations to this particular model. The procedure we will follow is: first, we find a best fit for the unpolarized valence distribution by comparing the LO and NLO evolution against the MRS [23] parametrization of the experimental data at $10\ GeV^2$. In doing that we fix the set of model parameters, e.g., radius of the bag and masses of the intermediate states. Using the virial theorem in the bag, where the bag itself carries the same energy as each of the three quarks confined in the bag, we roughly expect $M_s \simeq 750\ MeV - 150\ MeV$ and $M_v \simeq 750\ MeV + 50\ MeV$. But this is, of course, only an approximation and we have some freedom around these values. We also determine the scale μ at which the model is supposed to be valid. Using this set of parameters, we can then calculate the predictions of the model for other quantities. Here, as we have only the nonsinglet evolution for NLO, we will restrict ourselves to the polarized valence distribution of the proton and to the difference between the spin structure functions of the proton and neutron.

In LO it is found that for $R = 0.8\ fm$, $M_s = 0.55\ GeV$, $M_v = 0.75\ GeV$ and $\mu^2 = 0.0676\ GeV^2$, the unpolarized valence distribution of the proton in the bag model fits the MRS parametrization of the data. The result is displayed in Fig. 1 together with the bag model distribution at scale μ^2. The agreement is excellent with no significant discrepancy. However, at such a low scale the coupling constant is $\alpha(Q^2 = \mu^2) = 2.66$. This is a rather large value (as found in ref. [14]), and raises doubts about the whole procedure. In NLO the parameters used to fit the unpolarized valence distribution have the following values: $R = 0.8\ fm$, $M_s = 0.7\ GeV$, $M_v = 0.9\ GeV$ and $\mu^2 = 0.115\ GeV^2$. We notice that the radius for LO and NLO is the same but not the masses, being slightly bigger in the latter case. This is because in NLO one need not evolve as far, the scale μ^2 is larger and so the masses of the intermediate states need to be larger (the parton distributions peak at a larger x for smaller intermediate state masses). In Fig. 2 the NLO results are shown and again we see a very good agreement between the calculated valence distribution and the MRS parametrization of the data. What is remarkable now is the fact that the strong coupling constant drops to 0.77 at $\mu^2 = 0.115$ in NLO. Although this value is still large, it is much smaller than the LO case, making the evolution more reliable. Both LO and NLO

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moments are evolved to $Q^2 = 10 \ GeV^2$.

Once we have determined the parameters of the model, we are in the position to calculate other quantities. In Fig. 3 we plot the polarized u valence distribution for the bag model in LO and NLO against the preliminary experimental data [26]. There is no real difference between the LO and NLO curves (besides, of course, the mass parameters). Due to the large errors in the data it is impossible to conclude anything for the region $x \le 0.1$. But for $x \ge 0.1$ the theoretical curves certainly have the correct behaviour. Moreover, the polarized valence distributions are pure nonsinglet and we might expect that the bag model would give, in this case, a description of data as good as in the unpolarized case. This is because the problem with the polarized distributions is restricted to the singlet part of the structure function. The same considerations can be drawn from Fig. 4, where the polarized d valence distributions in LO and NLO are shown together with the data at $Q^2 = 10 \ GeV^2$.

Finally, in Fig. 5 we show the difference between the polarized structure functions of the proton and neutron (for which the integral is g_A). Only the theoretical values are presented.According to (8), in LO the moments of this quantity are just the moments of the parton distribution $\Delta u(x) - \Delta d(x)$ but in NLO we have to include, in addition to the NLO evolution of the parton distribution, the second order effects in the form of the corresponding Wilson coefficient (listed in the Appendix). The Wilson coefficient leads to a correction in the integral over the NLO curve in Fig. 5 of the form $1 - \alpha_s(Q^2)/\pi$. In practice, this reduces the integral from 1.32 in LO to 1.24 in NLO ($\alpha_s(Q^2)$ was calculated using the asymptotic approximation for equ. (6) and it has the value 0.191 at $Q^2 = 10 GeV^2$).

5   Conclusions

It is clear from the close agreement of the various quark distributions calculated in LO and NLO that the excellent results obtained in earlier studies based on the bag model were reliable. That the di-quark masses required (700 and 900 MeV) were so close to the values expected in the model (600 and 800 MeV - c.f. sect. 4) is very reassuring. Furthermore, the strong coupling constant in NLO is small enough that one can be quite confident in the convergence of the calculation.

It is important to be clear what has and has not been achieved. As most quark models (including the bag) are *not* derived from QCD one cannot calculate structure functions unambiguously at NLO. There is an inevitable limit to the accuracy of any quark model. What the present work has shown is that once the parameters of the model (e.g. μ^2, R, M_s, M_v) are fixed by the unpolarized valence distribution in either LO and NLO, the predictions for the spin dependent valence quark distributions are determined relatively unambiguously. This is an important result as it does give one some confidence in our ability to investigate problems like those mentioned in the introduction (nuclear EMC effect, spin crisis, etc.) within quark models.

As an immediate application we plan to estimate the effect of the pion cloud of the nucleon on the spin and flavour dependence of the bag structure functions [36].

A Appendix: Beta function, NS anomalous dimension and Wilson coeficient

We here list the coefficients appearing in the expansions (3)–(5). They can be found in refs. [28, 29, 31].

$$\beta_0 = \frac{11C_A - 4T_R N_f}{3}, \tag{A.1}$$

$$\beta_1 = \frac{34}{3}C_A^2 - 4\left(\frac{5}{3}C_A + C_F\right)T_R N_f, \tag{A.2}$$

$$\gamma^{(0)n} = 2C_F\left(-3 - \frac{2}{n(n+1)} + 4S_1(n)\right), \tag{A.3}$$

$$\gamma^{(1)n} = (C_F^2 - \frac{1}{2}C_F C_A)\left\{16S_1(n)\frac{2n+1}{n^2(n+1)^2} + 16\left[2S_1(n) - \frac{1}{n(n+1)}\right][S_2(n) - S_2'\left(\frac{n}{2}\right)]\right.$$

$$\left. +64\tilde{S}(n) + 24S_2(n) - 3 - 8S_3'\left(\frac{n}{2}\right) - 8\frac{3n^3+n^2-1}{n^3(n+1)^3} - 16(-1)^n\frac{2n^2+2n+1}{n^3(n+1)^3}\right\}$$

$$+C_F C_A\left\{S_1(n)\left[\frac{536}{9} + 8\frac{2n+1}{n^2(n+1)^2}\right] - 16S_1(n)S_2(n)\right.$$

$$\left. +S_2(n)\left[-\frac{52}{3} + \frac{8}{n(n+1)}\right] - \frac{43}{6} - 4\frac{151n^4+263n^3+97n^2+3n+9}{9n^3(n+1)^3}\right\}$$

$$+\frac{C_F N_f}{2}\left\{-\frac{160}{9}S_1(n) + \frac{32}{3}S_2(n) + \frac{4}{3} + 16\frac{11n^2+5n+-3}{9n^2(n+1)^2}\right\}, \tag{A.4}$$

$$F_n = C_F\left(-9 + \frac{1}{n} + \frac{2}{n+1} + \frac{2}{n^2} + 3S_1(n) - 4S_2(n) - \frac{2}{n(n+1)}S_1(n)\right.$$

$$\left. +2(S_1^2(n) + S_2(n))\right). \tag{A.5}$$

For the case of $SU(3)_c$, $C_F = \frac{4}{3}$, $C_A = 3$ and $T_R = \frac{1}{2}$. The parameter η is 1 or 2 for analytic continuation of odd or even momenta respectively. The functions $S(n)$ are given by:

$$S_i(n) = \sum_{j=1}^{n}\frac{1}{j^i}, \tag{A.6}$$

$$S_i'\left(\frac{n}{2}\right) = \frac{1+(-1)^\eta}{2}S_i\left(\frac{n}{2}\right) + \frac{1-(-1)^\eta}{2}S_i\left(\frac{n-1}{2}\right), \tag{A.7}$$

$$\tilde{S}(n) = \sum_{j=1}^{n} n\frac{(-1)^j}{j^2}S_1(j). \tag{A.8}$$

We use the inverse Laplace transformation of the IMSL library to make the inversion of expression (7) for the moments and extract the parton distributions. Once the moments are defined only for n even or odd, it is necessary to extend the validity of the equations for the whole interval of n. This is done through analytic continuation in which case the values of the anomalous dimensions and Wilson coefficients starting from even or odd n coincide. For this purpose, it is better to work with the forms of the $S(n)$ functions given in Appendix A of the 1990 paper of Glück et al. [22].

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Figure 1: Total valence distribution in the bag compared with the MRS [23] parametrization of the data in the $\overline{MS}$ scheme. The quark distributions are evolved in leading order QCD.

Figure 2: Total valence distribution in the bag compared with the MRS [23] parametrization of the data in the $\overline{MS}$ scheme. The quark distributions are evolved in next-to-leading order QCD.

Figure 3: Preliminary polarized valence data for the up quark distribution in the proton [26] compared with bag model predictions in leading order and next-to-leading order.

Figure 4: Preliminary polarized valence data for the down quark distribution in the proton [26] compared with bag model predictions in leading order and next-to-leading order.

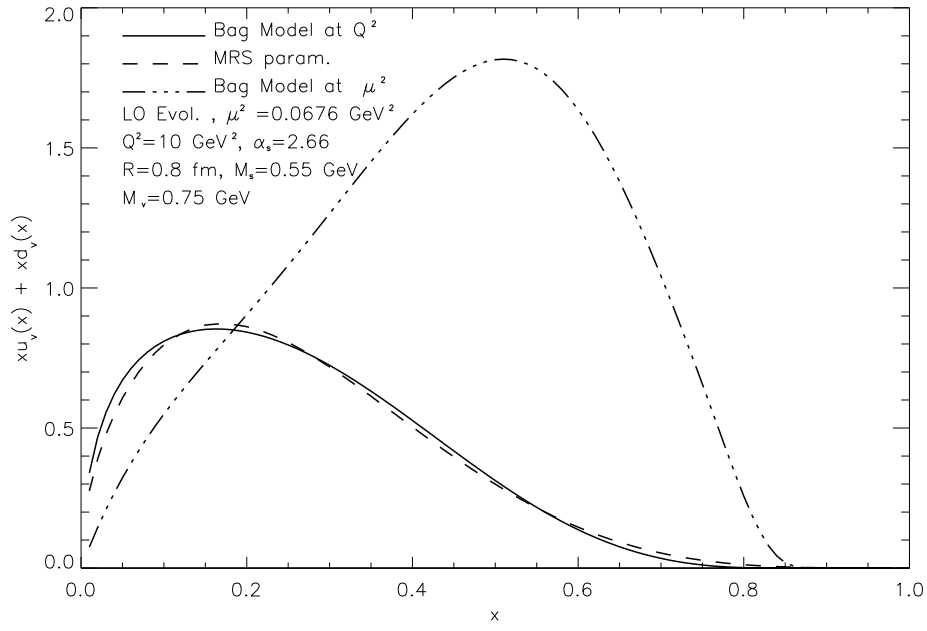Figure 5: Bag model prediction for the x dependence of the Bjorken sum rule in leading order and next-to-leading order QCD evolution.
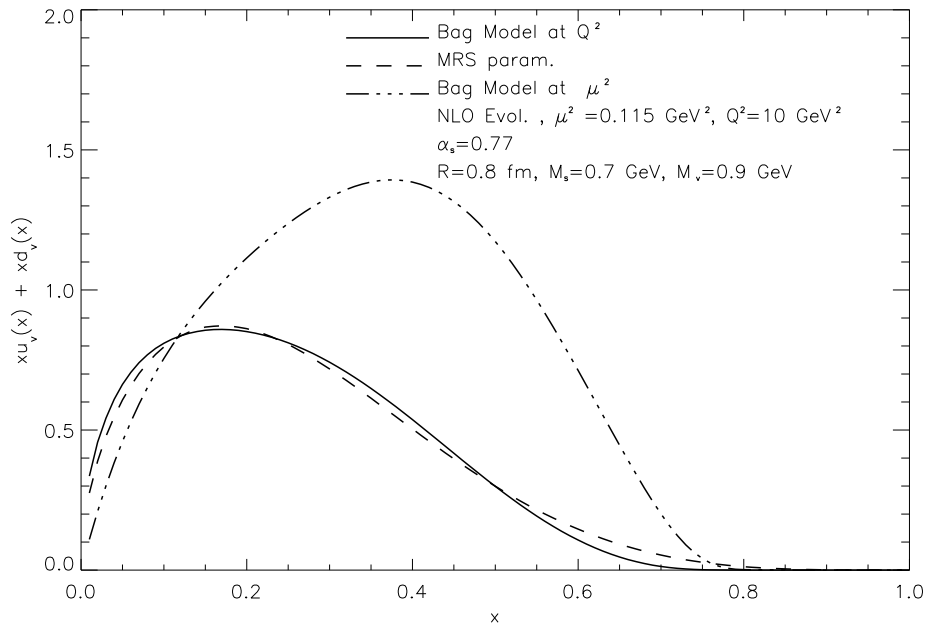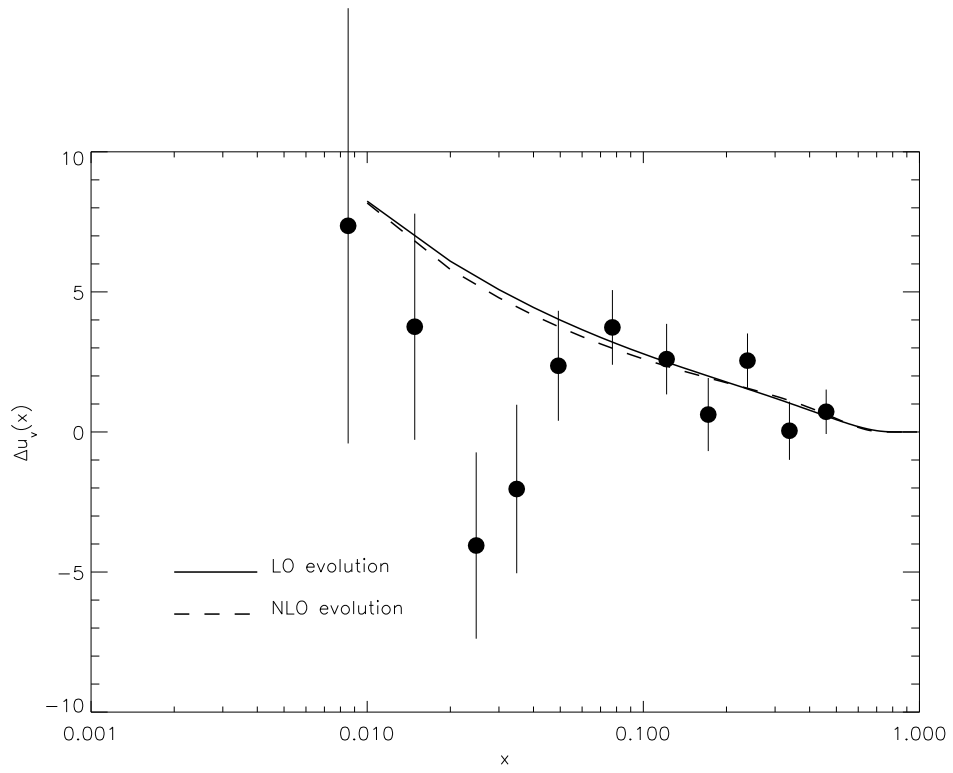
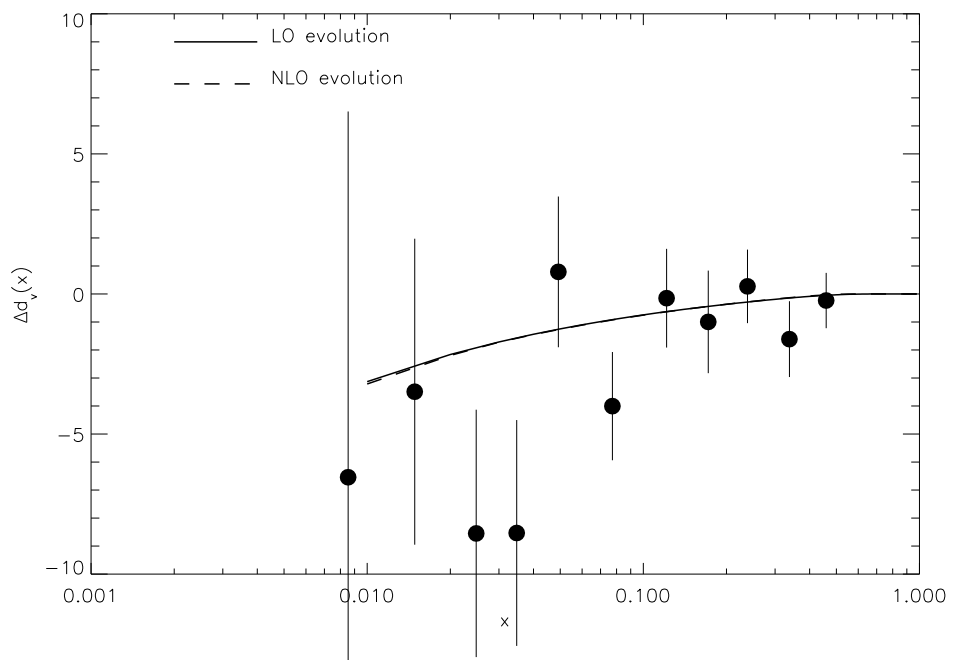Figure 1:

Figure 2:

Figure 3:

Figure 4:

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Figure 5: